Gas Flow in Porous Media – Turbulence or Thermodynamics

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Abstract

The flow of gas into a wellbore is calculated by the back-pressure or deliverability equation. The exponent n, which accounts for the deviation from the ideal Darcy flow, is usually attributed to turbulence in the reservoir. It is shown, that turbulence in gas wells only occurs in very extreme cases, and that the exponent n can be assigned to temperature changes during the expansion of gas. The exponent n is then similar to the reciprocal of the exponent in the equation of state, that describes the adiabatic reversible expansion of a gas. The deviation of Darcy flow, which is observed in gas wells, is not due to turbulence but to the change of temperature. The expression "non-Darcy flow" is misleading, as the flow does still follow Darcy's law.

1 Introduction

It was observed and is the common view in reservoir engineering, that Darcy's law is not valid for the flow of gas at high flow velocities, which occur during production in the vicinity of a gas well. This is expressed in the back pressure equation by the exponent n in equation

$$q = C \cdot (p_{ws}^2 - p_{wf}^2)^n \tag{1}$$

q is the volume flow rate, p_{ws} the reservoir pressure and p_{wf} the well flowing pressure. The exponent n is a measure for the deviation from the expected flow, a value of $n = 1$ would represent ideal flow.

It is believed that turbulence is responsible for this abnormality as described in many reservoir engineering text books. A detailed discussion of this theory is given by Slider [1], although he is still questioning the validity of this assumption.

Another attempt to describe the deviation of the expected ideal behaviour is the Forchheimer equation as e.g. described by Wattenbarger [2]. In the Forchheimer equation

$$\frac{dp}{dr} = \frac{\eta}{k} . \nu + F \cdot \rho \cdot \nu^2 \tag{2}$$

the Darcy equation (23) is extended by a term, which is dominated by the square of the flow-velocity. ρ is the fluid density and F is a "turbulence factor".

In the following it will be analysed, when turbulence occurs in gas flows at the conditions encountered during the production of a gas well. The empirical "back pressure-equation" (1) will then be derived, considering thermodynamics of the gas flow.

2 Occurrence of Turbulence in Single Phase Flow in Porous Media

During flow of liquids in long pipes it was observed, that at higher velocities the Hagen-Poiseuille equation for the calculation of the pressure drop is no longer valid and has to be replaced by another law (Prandtl [3]).

The pressure drop increases significantly as soon as a critical value in flow velocity is reached. Instead of laminar flow the observed streamlines are disturbed. This is usually called turbulent flow. In general turbulent flow occurs at Reynolds numbers greater than 2320. The Reynolds number is defined as

$$\mathrm{Re} = \rho \cdot d \cdot \nu / \eta \tag{3}$$

where ρ is the density of the fluid, d the length of the body (in pipes the pipe diameter), v the flow velocity and η the viscosity.

For the flow in porous media there are different approaches for the calculation of the Reynolds number.

In porous media the introduction of the term turbulence means, that at a particular flow velocity Darcy's law is no longer valid, i. e. that the pressure drop is higher than calculated by Darcy's law. This behaviour may be regarded analogue to the flow in pipes, where the transition from laminar to turbulent flow occurs at Reynolds numbers greater than 2320.

In both cases this means, that the pressure drop cannot be calculated exactly, but empirical corrections have to be applied (friction factors).

Analogue to the flow in pipes it was attempted to describe the point at which the Darcy equation becomes invalid, with Reynolds numbers [4, 5]. In flow through a porous medium it is not possible to exactly specify the pore diameter as well as the flow velocity in a pore. To account for these circumstances different approaches were chosen in literature. These and another own derived approach are described in the following.

2.1 Calculation by grain size (Muskat [4])

This approach uses for the calculation of a Reynolds number the average grain size for d and the Darcy velocity ν_D:

If Darcy's law for linear flow is written as

$$\nu_D = \frac{k}{\eta} \cdot \frac{\Delta p}{l} \tag{4}$$

where, ν_D is defined as the volume flow rate q divided by the area of the cross section of the porous body,

$$\nu_D = \frac{q}{A} \tag{5}$$

Muskat [4] derives the transition to turbulence from flow experiments at Reynolds numbers between 1 and 10

$$d \cdot \nu_D \cdot \rho / \mu > 10 \tag{6}$$

Note that in these measurements, which were published in 1933, the deviations from Darcy's law were only observed in experiments performed with gas, but not at measurements that were performed at similar flow rates with water.

2.2 Start of turbulent flow after Dullien [5]

Dullien [5] uses the Darcy velocity and for d the square root of the permeability $\sqrt{k}$ for the calculation of the Reynolds number

$$\mathrm{Re} = \rho \cdot \sqrt{k} \cdot \nu_D / \eta \tag{7}$$

His criterion for the start of turbulent flow is

$$\mathrm{Re} > 1 / c \tag{8}$$

where c varies between 0.075 and 0.5 or 1/c between 2 and 13.

2.3 Calculation of the Reynolds number using the hydraulic radius concept

For the description of the pore geometry the definition of the hydraulic diameter d_H is helpful:

$$d_H = \frac{4V_F}{S_V} \tag{9}$$

where V_F is the volume occupied by the fluid

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and S_V is the inner surface of the solid body. V shall be the volume of the entire body and V_B the bulk volume. In the following it is supposed, that V_F is the pore volume V_p. The specific surface of the porous body is given by

$$S_o = \frac{S_V}{V_B} \tag{10}$$

and thus for the hydraulic diameter of a porous body may be written as follows:

$$d_H = \frac{4V_P}{S_V} = \frac{4\cdot\varepsilon\cdot V}{S_o\cdot V_B} = \frac{4\cdot\varepsilon\cdot V}{S_o(1-\varepsilon)V} = \frac{4\varepsilon}{S_o(1-\varepsilon)} \tag{11}$$

where ε is the porosity.

Kozeny [6] and Carman [7, 8] derived an equation that calculates the permeability k of a well sorted sand

$$k = \frac{1}{k_o T S_0^2}\cdot\frac{\varepsilon^3}{(1-\varepsilon)^2} \tag{12}$$

T is the tortuosity factor defined as the square of the ratio of the actual flow path L_ε and the length of the porous body and k_o is a shape factor, which is usually set to 2.

$$T = \left(\frac{L_\varepsilon}{L}\right)^2 \tag{13}$$

Substituting S_o from eq. (12) into eq. (11) we get for the hydraulic diameter

$$d_H = \sqrt{\frac{16\cdot k_o\cdot T\cdot k}{\varepsilon}} \tag{14}$$

The flow velocity in the pore space is given in the following equation.

$$\nu = \nu_D\frac{\sqrt{T}}{\varepsilon} \tag{15}$$

(v_D = Darcy-velocity)

With the shape factor k_o=2 and T=1

$$d_H = \sqrt{32\cdot\frac{k}{\varepsilon}} \tag{16}$$

and

$$\nu = \frac{\nu_D}{\varepsilon} \tag{17}$$

If we use these definitions for d and v to calculate the Reynolds number in a porous medium, we can use the same criterion for the onset of turbulent flow as in a pipe.

$$\mathrm{Re} = \rho\cdot d\cdot\nu/\mu > 2320 \qquad 18)$$

Table 1 **Occurrence of turbulence, calculated after different methods**

Pressure:	120 bar
Temperature:	40 °C
Well diameter:	9 5/8″
Flow rate:	30,000 $m^3(V_n)/h$
eff. Thickness:	10 m
Viskosity:	0.015 mPas
Density:	0.73 $kg/m^3(V_n)$
Permeability:	500 mD
Porosity:	20 %
Grain diameter:	100 µm
Hydraulic radius:	4.47 µm

Radius m	vD m/s	Re after MUSKAT	Re after d_H	Re after DULLIEN	1/Re DULLIEN	Turbulent acc. to MUSKAT	Turbulent acc. to d_H.	Turbulent acc. to DULLIEN
0.0714375	$1.55\cdot10^{-2}$	9.04	2.02	$6.39\cdot10^{-2}$	15.65	no /?	no	no
0.1	$1.11\cdot10^{-2}$	6.46	1.44	$4.56\cdot10^{-2}$	21.91	no/?	no	no
0.2	$5.53\cdot10^{-3}$	3.23	$7.22\cdot10^{-1}$	$2.28\cdot10^{-2}$	43.81	no	no	no
0.3	$3.68\cdot10^{-3}$	2.15	$4.81\cdot10^{-1}$	$1.52\cdot10^{-2}$	65.72	no	no	no
0.4	$2.76\cdot10^{-3}$	1.61	$3.61\cdot10^{-1}$	$1.14\cdot10^{-2}$	87.62	no	no	no
1	$1.11\cdot10^{-3}$	$6.46\cdot10^{-1}$	$1.44\cdot10^{-1}$	$4.56\cdot10^{-3}$	219.06	no	no	no
2	$5.53\cdot10^{-4}$	$3.23\cdot10^{-1}$	$7.22\cdot10^{-2}$	$2.28\cdot10^{-3}$	438.12	no	no	no
3	$3.68\cdot10^{-4}$	$2.15\cdot10^{-1}$	$4.81\cdot10^{-2}$	$1.52\cdot10^{-3}$	657.18	no	no	no
5	$2.21\cdot10^{-4}$	$1.29\cdot10^{-1}$	$2.89\cdot10^{-2}$	$9.13\cdot10^{-4}$	1095.30	no	no	no
10	$1.11\cdot10^{-4}$	$6.46\cdot10^{-2}$	$1.44\cdot10^{-2}$	$4.56\cdot10^{-4}$	2190.60	no	no	no

3 Example Calculation of Turbulent Flow for a Sandstone

Table 1 shows the results of calculations of the Reynolds number for the different methods described above. The examples are for a gas production well in a reservoir having a thickness of 10 m. The well diameter is 9 5/8″, which is a typical diameter of a gas producing well. The permeability of the reservoir is 500 mD, the porosity 20%. The calculations are made for a production rate of 30,000 $m^3(V_n)/h$, which is also a typical rate for these reservoir conditions. The reservoir pressure is 120 bar. For the calculations according to Muskat a grain diameter of 100 µm was chosen, which is representative for this reservoir.

The calculations show, that the flow around a wellbore at regular production conditions of a gas well is not turbulent. Even, when applying the criteria of Muskat or Dullien turbulence would occur at grain sizes greater than 0.5 mm and production rates above 70,000 $m^3(V_n)/h$. Such grain sizes are normally found in gravel packs, but not in reservoir sands. It may be concluded, that turbulent flow in the pore space around a well bore during gas production does not occur.

Therefore it is not likely, that the deviation from normal Darcy flow, as described in eq. (1) is caused by turbulence. In gas reservoirs the permeabilities or grain diameters are usually smaller than as taken in the example in Table 1, which means that d in the calculation for the Reynolds number is smaller and thus the onset of turbulence even later.

4 The pVT Behaviour of Gases

The pressure-volume-behaviour of gases is described by eq. (19).

$$p\cdot V = z\cdot n\cdot R\cdot T \tag{19}$$

where p is the pressure, V the volume and T the temperature of the gas, n is the number of moles, z is a factor, which describes the deviation from the behaviour of an ideal gas. R is the gas constant. The z-factor can be calculated for every pressure and temperature (above the critical point and values relevant for the conditions discussed here), if the gas composition is known, from different correlations. Eq. (19) is valid as long as the volume and pressure changes are isotherm. If e. g. a gas expands, work has to be performed by the gas. The gas will take heat from its environment in the same amount as of this work. In this case the change of state is isotherm. If no heat transfer occurs, the temperature of the gas will change. In this adiabatic case, where no heat transfer from the environment occurs, the change in volume and pressure is then described by

$$V_2 = \left(\frac{p_1}{p_2}\right)^{1/\kappa}\cdot V_1 \tag{20}$$

where $\kappa = c_p/c_v$, the ratio of the specific heat capacities at constant pressure and volume, is the adiabatic exponent. The change of state is reversible, i. e. the gas state is always in equilibrium.

or

$$p\cdot V^\kappa = const \tag{21}$$

The isothermal case will occur in a gas reservoir, where the flow rate of gas is low and heat transfer from the surrounding rock is granted. In the vicinity of a well bore, where high flow rates occur this may not be applied. The conditions will be more adiabatic and the temperature of the gas may change significantly.

5 Derivation of the "Back Pressure Equation"

The Darcy equation for the radial flow of incompressible fluids may be written as

$$\eta\cdot\frac{q}{A} = k\cdot\frac{dp}{dr} \tag{22}$$

or

$$\frac{dp}{dr} = \frac{k}{\eta}\nu_D \tag{23}$$

where η is the viscosity, q the volume flow

Table 2 **Parameters and results of well tests**

Well	Rate m^3(Vn)/h	T_1 °C	p_1 bar	p_2 bar	Measured temperature, °C	n	1/n	?	c_v/R	$T_{2\,(calc)}$ °C (reversible)	$T_{2\,(calc)}$ °C (irreversible)
A	22 000	40.2	124.40	120.90	35.3	0.570	1.754	1.47	17.69	37.35	37.38
	29 000	40.2	124.40	118.60	34.8	0.570	1.754	1.47	17.69	35.45	37.38
	35 500	40.2	124.40	116.10	34.2	0.570	1.754	1.47	17.69	33.36	33.52
	50 000	40.2	124.40	109.00	32.5	0.570	1.754	1.47	17.69	27.24	27.80
B	10 000	44.3	126.13	120.00	38.3	0.479	2.088	1.47	17.69	39.30	39.39
	17 000	44.3	126.13	110.07	36.0	0.479	2.088	1.47	17.69	30.79	31.40
	19 500	44.3	126.13	104.37	35.4	0.479	2.088	1.47	17.69	25.67	26.81
C	9 000	40.5	126.90	126.15	37.5	0.740	1.351	1.47	17.69	39.91	39.91
	20 000	40.5	126.90	124.32	36.5	0.740	1.351	1.47	17.69	38.45	38.46
	30 000	40.5	126.90	122.28	35.2	0.740	1.351	1.47	17.69	36.80	36.85
D	24 200	41.4	125.68	123.67	38.4	0.639	1.565	1.47	17.69	39.74	39.75
	28 800	41.4	125.68	122.83	38.4	0.639	1.565	1.47	17.69	39.06	39.08
	40 900	41.4	125.68	120.88	38.3	0.639	1.565	1.47	17.69	37.47	37.52
	50 400	41.4	125.68	118.87	38.2	0.639	1.565	1.47	17.69	35.81	35.91
E	25 000	40.8	123.58	121.67	38.9	0.592	1.689	1.47	17.69	39.24	39.25
	28 000	40.8	123.58	120.60	38.9	0.592	1.689	1.47	17.69	38.36	38.38
	37 000	40.8	123.58	119.17	38.8	0.592	1.689	1.47	17.69	37.17	37.22
	55 000	40.8	123.58	115.88	38.4	0.592	1.689	1.47	17.69	34.41	34.55

The calculated temperatures are not corrected with z-factors.

rate, A the cross sectional area, k the permeability, p the pressure and r the radius.

For a compressible fluid like gas, the density is not constant, but a function of the pressure p: $\rho = \rho(p)$. For a perfect gas the flow rate may be written as

$$q(p) = \frac{1}{p} q \tag{24}$$

Then eq. (22) becomes with $A = 2\pi rh$ (h = reservoir thickness)

$$\frac{\eta \cdot q}{2\pi h} \cdot \frac{1}{r} dr = k \cdot p\,dp \tag{25}$$

the integration gives

$$\frac{\eta \cdot q}{2\pi h} \cdot \int_{rw}^{re} \frac{1}{r} dr = k \cdot \int_{pwf}^{ps} p\,dp \tag{26}$$

$$q = \frac{\pi kh}{\eta \cdot ln(re/rw)} \cdot \left(p_s^2 - p_{wf}^2\right) \tag{27}$$

or

$$q = C' \cdot \left(p_s^2 - p_{wf}^2\right) \tag{28}$$

Eq. (28) should describe the isothermal case of gas flow.

In the case of adiabatic flow we may write

$$q(p) = \frac{1}{p} \cdot q^{\kappa} \tag{29}$$

inserting this into the Darcy equation (25) we obtain

$$q = C' \cdot \left(p_s^2 - p_{wf}^2\right)^{1/\kappa} \tag{30}$$

with

$$1/\kappa = n \tag{31}$$

and converting C' to standard conditions we obtain

$$q = C \cdot \left(p_{ws}^2 - p_{wf}^2\right)^n \tag{32}$$

with

$$C = \left(\frac{p \cdot z_s}{p_s \cdot z} \cdot \frac{\pi \cdot k \cdot h}{\eta \cdot \ln\left(r_e / r_w\right)}\right) \tag{33}$$

For a perfect gas $\kappa = 5/3$ and therefore n = 0.6, for methane =1.308 und n = 0.765, for ethane =1.188 und n = 0.84. These values would match perfectly to those observed for n in gas well testing.

6 Example of a Gas Well Test – Conclusions

In Figure 1 the pressure and the temperature development during a gas well test (flow after flow) is given.

The test parameters are given, together with some other test results, in Table 2. In Table 2 also the measured temperatures and calculated temperatures are listed. The temperature in the well for an adiabatic reversible expansion can be calculated using eq. (34)

$$T_2 = T_1 \left(\frac{p_2 \cdot z_1}{p_1 \cdot z_2}\right)^{(\kappa - 1)/\kappa} \tag{34}$$

The volume is constant and the pressures may be corrected with the z-factors at the particular pressure and temperature. T_1 is the reservoir temperature and p_1 the shut in pressure, p_2 and T_2 the well flowing pressure and temperature. For the calculation the temperature should be in Kelvin.

For the calculation of an irreversible expansion eq. (35) is used.

$$T_2 = \left(\frac{1}{1 + c_v / R}\right) \cdot \left(\frac{p_2}{p_1} + \frac{c_v}{R}\right) \cdot T_1 \tag{35}$$

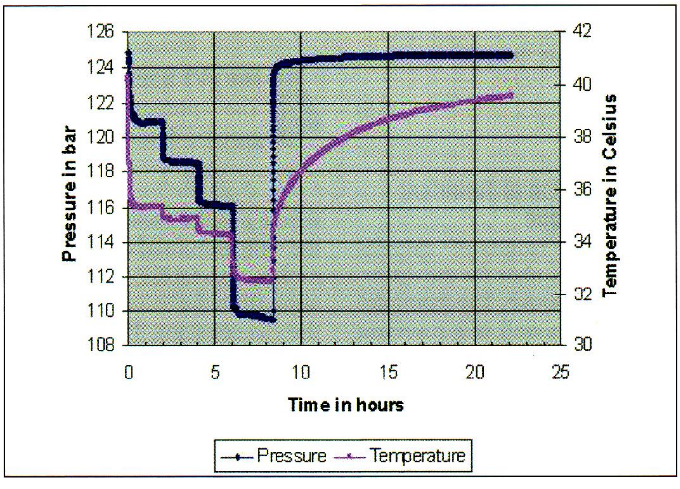

Fig. 1 Well test, pressure and temperature development

where c_v is calculated from κ using

$$c_v = \frac{R}{\kappa - 1} \tag{36}$$

It is evident, that the deviation of Darcy flow, which is observed in gas wells, is not due to turbulence but to the non isothermal changes of state. The expression "non-Darcy flow" is misleading, as the flow of course still follows Darcy's law.

The above derivations for the back pressure equations were made for the idealized cases of adiabatic reversible changes of state and for a perfect gas. In a real case in a wellbore the changes of state may become irreversible and the idealized parameters may not be valid. Furthermore heat flow from the reservoir will occur and warm up the gas. This is indicated by the results obtained from the calculations in Table 2. The temperature values calculated for an adiabatic change of state are in the same range as those observed, but are too low at higher rates. It should be taken into consideration, that these calculations were made for idealised conditions.

Conclusion: More attention should be paid to thermodynamics in gas production. This may also apply to other reservoir engineering problems as e.g. in gas storage operations, where also pressure developments have been observed, which cannot be described by normal isothermal reservoir engineering approaches. It may be assumed, that this is also due to temperature changes in the reservoir, which influence pressure significantly, especially as temperature equilibration (heat flow from the reservoir rock) is not very rapid, but more slowly as e.g. the pressure build-up as shown in Fig. 1. This indicates, that also for well test evaluations from pressure build-up, the non iso- thermal behaviour in gas flow should be considered.

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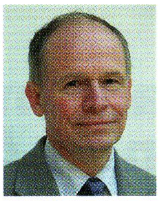

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